Numerical simulation of fluid flow and seismic wave propagation as a tool to monitor CO$_2$ sequestration

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Introduction

- Fossil-fuel combustion generates carbon dioxide (CO$_2$), which is mainly discharged into the atmosphere, increasing the atmosphere temperature (greenhouse effect).
- To minimize climate change impacts, geological sequestration of CO$_2$ is an immediate option.
- Geologic sequestration involves injecting CO$_2$ into a target geologic formation at depths typically >1000 m where pressure and temperature are above the critical point for CO$_2$ (31.6°C, 7.38 MPa).
Introduction

- First industrial scale CO\textsubscript{2} injection project: Sleipner gas field (North Sea).
- CO\textsubscript{2} is separated from natural gas produced and is currently being injected into the Utsira Sand, a saline aquifer.
- Injection started in 1996 and is planned to continue for about 20 years, at a rate of about one million tonnes per year.
Objective

- Very little is known about the effectiveness of CO\textsubscript{2} sequestration over very long periods of time.
- Therefore, the objective of this work is to introduce a methodology that integrates numerical simulation of CO\textsubscript{2} - brine flow and seismic wave propagation to model and monitor CO\textsubscript{2} injection. This methodology will help to analyze if underground storage is a safe and verifiable technology in the long term.
Presentation Outline:

• Present the two-phase fluid flow equations used to simulate CO$_2$ injection.


• Present the numerical procedures employed to compute approximate solutions to the model.

• Show numerical simulations of CO$_2$ injection and time-lapse seismics to monitor the migration and dispersal of CO$_2$ after injection in the Utsira formation in the Sleipner field.
The Black-Oil formulation

• The simultaneous flow of brine and CO$_2$ is described by the well-known Black-Oil formulation applied to two-phase, two component fluid flow.

• In this model, CO$_2$ may dissolve in the brine but the brine is not allowed to vaporize into the CO$_2$ phase.

• This formulation uses, as a simplified thermodynamic model, the following PVT data:

  \[ R_s : \text{CO}_2 \text{ solubility in brine}, \]
  \[ B_{CO_2} : \text{CO}_2 \text{ formation volume factor}, \]
  \[ B_b : \text{brine formation volume factor}. \]

They are determined using the Hassanzadeh’s correlations.
The Black-Oil formulation of two-phase flow in porous media

The mass conservation equations are:

\[
- \nabla \cdot \left( \frac{1}{B_{CO_2}} \nu_{CO_2} + \frac{R_s}{B_b} \nu_b \right) + q_{CO_2} = \partial \left[ \phi \left( \frac{1}{B_{CO_2}} S_{CO_2} + \frac{R_s}{B_b} S_b \right) \right] / \partial t,
\]

\[
- \nabla \cdot \left( \frac{1}{B_b} \nu_b \right) + q_b = \partial \left[ \phi \left( \frac{1}{B_b} S_b \right) \right] / \partial t
\]

\(S_\beta\): phase \(\beta\) saturation,

\(\nu_\beta\): Darcy velocity of phase \(\beta\)

\(q_\beta\): flow rate per unit volume

\(\phi\): porosity
The Black-Oil formulation of two-phase flow in porous media

The momentum balance for the fluids is given by Darcy’s Law:

\[
\begin{align*}
\nu_b &= -\kappa \frac{\kappa_r b}{\mu_b} (\nabla p_b - \rho_b g \nabla D), \\
\nu_{CO_2} &= -\kappa \frac{\kappa_r CO_2}{\mu_{CO_2}} (\nabla p_{CO_2} - \rho_{CO_2} g \nabla D),
\end{align*}
\]

\( p_\beta \): phase \( \beta \) pressure

\( \mu_\beta \): phase \( \beta \) viscosity,

\( \kappa \): absolute permeability

\( \kappa_r \beta \): phase \( \beta \) relative permeability

Replacing Darcy’s Law into mass conservation equations:
The Black-Oil formulation of two-phase flow in porous media

\[ \nabla \cdot \left( k \left( \frac{k_{rCO2}}{B_{CO2} \mu_{CO2}} \nabla p_{CO2} - \rho_{CO2} g \nabla D \right) + \frac{R_s k_{rb}}{B_b \mu_b} \left( \nabla p_b - \rho_b g \nabla D \right) \right) + \nabla \cdot \left( k \frac{k_{rCO2}}{B_{CO2} \mu_{CO2}} \nabla p_{CO2} - \rho_{CO2} g \nabla D \right) = \frac{\partial}{\partial t} \phi \left( \frac{S_{CO2}}{B_{CO2}} + \frac{R_s S_b}{B_b} \right), \]

Two algebraic equations complete the system:

\[ S_b + S_{CO2} = 1, \quad p_{CO2} - p_b = P_C(S_b) \]
Numerical solution of the Black-Oil formulation of two-phase flow in porous media.

The unknowns for the Black-Oil fluid-flow model are the fluid pressures \( p_{CO2} \), \( p_b \) and the saturations \( S_{CO2} \), \( S_b \) for the CO\(_2\) and brine phases.

They were computed using the public domain software BOAST, which solves the differential equations applying IMPES, a finite difference technique.
Biot’s Equations of Motion.

\[-\omega^2 \rho_b u^{(s)} - \omega^2 \rho_f u^{(f)} - \nabla \cdot \sigma(u) = F^{(s)}\]

\[-\omega^2 \rho_f u^{(s)} - \omega^2 g u^{(f)} + i \omega b u^{(f)} + \nabla p_f(u) = F^{(f)} .\]

\[
\rho_b = (1 - \phi) \rho_s + \phi \rho_f , \quad g = \frac{s \rho_f}{\phi} , \quad b = \frac{\mu}{\kappa} .
\]

\( \rho_f , \rho_s \): mass densities of the fluids and the solid grains,

\( s \): Tortuosity factor.
The Biot model for wave propagation in fluid saturated porous media

\( u^{(s)}, \tilde{u}^{(f)} \): time Fourier transform (FT) of the averaged displacement vectors of the solid and fluid phases, respectively.

\[ u^{(f)} = \phi(\tilde{u}^{(f)} - u^{(s)}), \quad u = (u^{(s)}, u^{(f)}). \]

\( \sigma_{ij} \): FT of the stress tensor of the bulk material,

\( p_f \): FT of the fluid pressure,

\( \varepsilon_{ij} \): FT of the strain tensor of the solid.
Constitutive Relations

\[
\sigma_{ij}(u) = 2N \varepsilon_{ij}(u^{(s)}) + \delta_{ij}(\lambda_c \nabla \cdot u^{(s)} + \alpha K_{av} \nabla \cdot u^{(f)}),
\]

\[
p_f(u) = -\alpha K_{av} \nabla \cdot u^{(s)} - K_{av} \nabla \cdot u^{(f)},
\]

In the elastic case, the coefficients in the stress-strain relations can be obtained in terms of:

- \(N\): shear modulus of the dry matrix
- \(K_m\): bulk modulus of the dry matrix
- \(K_s\): bulk modulus of the solid grains
- \(K_f\): bulk modulus of the saturant fluid
- \(\phi\): effective porosity
Constitutive Relations

\[
\alpha = 1 - \frac{K_m}{K_s},
\]

\[
K_{av} = \left[ \frac{\alpha - \phi}{K_s} + \frac{\phi}{K_f} \right]^{-1}.
\]

\[
K_c = K_m + \alpha^2 K_{av}
\]

\[
\lambda_c = K_c - \frac{2}{3} N,
\]

\(K_c\): Gassmann’s (closed) bulk modulus of the saturated material.
Weak Formulation with an Absorbing Boundary Condition. 2D Case.

\( \Omega \): rectangular domain, \( \mathcal{V} = [H^1(\Omega)]^2 \times H(\text{div}; \Omega) \).

Testing Biot’s equation of motion with \( v \in \mathcal{V} \) we get the weak form:

\[
-\omega^2 (P_{u,v}) + i\omega (B_{u,v}) + A(u,v) + i\omega \langle D_{u,v} \rangle = (F,v),
\]

\[
v = (v^{(s)}, v^{(f)})^t \in \mathcal{V},
\]

\[
A(u,v) = \sum_{l,m} (\sigma_{lm}(u), \varepsilon_{lm}(v^{(s)})) - (p_{f}(u), \nabla \cdot v^{(f)}) \), \quad u, v \in \mathcal{V}.
\]

\[
P = \begin{pmatrix}
\rho_b I & \rho_f I \\
\rho_f I & g I
\end{pmatrix}, \quad 
B = \begin{pmatrix}
0 I & 0 I \\
0 I & b I
\end{pmatrix}, \quad 
F = (F^{(s)}, F^{(f)}).
\]

\(I\): identity matrix in \( R^2 \times 2 \). \(D\): a positive definite matrix.
Finite element spaces for Biot’s equations of motion.

Rectangular Elements.

\[ \mathcal{V}^h = \mathcal{N} C^h \times \mathcal{N} C^h \times \mathcal{W}^h \]

\( \mathcal{N} C^h \): a nonconforming finite element space used to approximate each component of the solid displacement.

\( \mathcal{W}^h \): finite element space used to approximate the fluid displacement.

Local degrees of freedom
Idealized model of the Utsira formation

We consider an idealized geometrical and physical model of the Utsira formation consisting of 5 regions. We modeled the cases when $\Omega_4$ is either shale seal layer or a fractured shale seal layer.

The Biot model assumes a single-phase fluid. Effective fluid density, viscosity and bulk modulus were obtained using the properties of the CO$_2$ and brine weighted by the corresponding saturations computed by the flow simulator. The medium is excited with a compressional point source located at $x= 400$ m, $z= 710$ m.
Idealized model of the Utsira formation

The injection point is located at $x = 400$ m, $z = 1060$ m.
CO$_2$ saturation after 5 years of injection. Case when $\Omega_4$ is a 2 m shale seal layer.

The injection point is located at $x=400$ m, $z=1060$ m.
Traces of particle velocity of the solid phase before and after 5 years of CO2 injection. $\Omega_4$ is a shale seal layer.

![Diagram showing particle velocity traces](image)

- **Direct wave**
- **Reflection from CO2 accumulation**
- **Traces from receiver at $x = 300$ m, $z = 10$ m**
Time histories measured near the surface before and after 5 years of CO2 injection. $\Omega_4$ is a shale seal layer.

The first reflection in both figures is due to the direct wave coming from the point source. The second reflection is generated by CO$_2$ accumulations below the thin shale layer at depth $z = 940$ m. Before injection, this seal layer can not be observed.
Vertical component of the solid phase velocity at 170 ms before and after 5 years of CO2 injection. $\Omega_4$ is a shale seal layer.

Left: before injection; right: after 5 years of CO$_2$ injection. At 170 ms the wave front generated by the compressional point source located at $x=400$ m, $z=710$ m is arriving at the thin shale layer at $z=940$ m.
Vertical component of the solid phase velocity at 200 ms before and after 5 years of CO2 injection. $\Omega_4$ is a shale seal layer.

Left: before injection; right: after 5 years of CO2 injection. At 200 ms the waves generated by the point source have generated reflected and transmitted waves due to the CO2 accumulation below the thin shale layer at $z = 940$ m.
Vertical component of the solid phase velocity at 260 ms before and after 5 years of CO2 injection. Case when $\Omega_4$ is a shale seal layer.

Left: before injection; right: after 5 years of CO$_2$ injection. The reflected and transmitted waves from the CO$_2$ accumulation below the thin shale layer at $z = 940$ m are clearly observed.
CO$_2$ saturation after 3 and 5 years of injection. Case when \( \Omega_4 \) is a fractured shale seal layer.

The injection point is located at \( x = 400 \) m, \( z = 1060 \) m.
Vertical component of the solid phase velocity at 110, 140, 170 and 200 ms after 5 years of CO2 injection. $\Omega_4$ is a fractured shale seal layer.

The reflected and transmitted waves from the CO$_2$ accumulation below the top of the Utsira and the thin shale layer at $z = 940$ m are clearly observed.
Time histories measured near the surface before and after 5 years of CO2 injection. Case when $\Omega_4$ is a fractured shale seal layer.

The 1rst reflection is due to the direct wave coming from the point source. The 2nd and 3rd reflections are generated by CO$_2$ accumulations below the top of the Utsira and the fractured shale seal layer.
Traces of particle velocity of the solid phase before and after 5 years of CO2 injection for both cases of $\Omega_4$. 

- Reflection from CO2 below top of Utsira.
- Reflection from CO2 below thin seal.
- Direct wave.
- Traces at $x=150$ m, $z=10$ m.
$\Omega_4$ is a shale seal layer and there is an additional layer of small permeability.

There is CO$_2$ accumulation in the low permeability layer.
Traces of particle velocity of the solid phase after 5 years of CO2 injection for both cases, with and without the low permeability layer
CONCLUSIONS

• We integrate numerical simulators of the CO$_2$-brine flow and seismic wave propagation to model and monitor CO$_2$ storage.

• Numerical examples show the effectiveness of this methodology to detect the spatio-temporal distribution of CO$_2$

• This methodology constitutes an important tool to monitor the migration and dispersal of the CO$_2$ plume and to analyze storage integrity, providing early warning should any leakage occurs.